EXERCISE 8.1 [PAGES 111 - 112]

Exercise 8.1 | Q 1.1 | Page 111

Examine the continuity of $f(x) = x^3 + 2x^2 - x - 2$ at x = -2.

SOLUTION

 $f(x) = x^3 + 2x^2 - x - 2$ Here f(x) is a polynomial function and hence it is continuous for all x ∈ R. \therefore f(x) is continuous at x = -2.

Exercise 8.1 | Q 1.2 | Page 111

Examine the continuity of f(x) = $rac{x^2-9}{x-3}$ on R.

SOLUTION

$$f(x) = \frac{x^2 - 9}{x - 3}; x \in R$$

f(x) is a rational function and is continuous for all $x \in R$, except at the points where denominator becomes zero. Here, denominator x - 3 = 0 when x = 3.

 \therefore Function f is continuous for all $x \in R$, except at x = 3, where it is not defined.

Exercise 8.1 | Q 2.1 | Page 111

Examine whether the function is continuous at the points indicated against them.

 $\begin{aligned} f(x) &= x^3 - 2x + 1, & \text{for } x \leq 2 \\ &= 3x - 2, & \text{for } x > 2, \text{ at } x = 2. \end{aligned}$

SOLUTION

$$\lim_{x \to 2^{-}} f(x) = \lim_{x \to 2^{-}} (x^3 - 2x + 1)$$
$$= = (2)^3 - 2(2) + 1 = 5$$



 $\lim_{x \to 2^+} f(x) = \lim_{x \to 2^+} (3x - 2)$ = 3(2) - 2 = 4

$$\therefore \lim_{x
ightarrow 2^-} f(x)
eq \lim_{x
ightarrow 2^+} f(x)$$

 \therefore Function f is discontinuous at x = 2

Exercise 8.1 | Q 2.2 | Page 111

Examine whether the function is continuous at the points indicated against them.

f(x) =
$$\frac{x^2 + 18x - 19}{x - 1}$$
 for x ≠ 1
= 20 for x = 1, at x = 1

SOLUTION

$$\lim_{x \to 1} f(x) = \lim_{x \to 1} \frac{x^2 + 18x - 19}{x - 1}$$

=
$$\lim_{x \to 1} \frac{x^2 + 19x - x - 19}{x - 1}$$

=
$$\lim_{x \to 1} \frac{x(x + 19) - 1(x + 19)}{x - 1}$$

=
$$\lim_{x \to 1} \frac{(x - 1)(x + 19)}{(x - 1)}$$

=
$$\lim_{x \to 1} (x + 19) \dots [\because x \to 1, \therefore x \neq 1, \therefore x - 1 \neq 0]$$

=
$$1 + 19 = 20$$

Also, f(1) = 20
$$\therefore \lim_{x \to 1} f(x) = f(1)$$

$$\therefore f(x) \text{ is continuous at } x = 1$$

Exercise 8.1 | Q 3.1 | Page 112

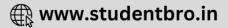
Test the continuity of the following function at the points indicated against them.

$$f(x) = \frac{\sqrt{x-1} - (x-1)^{\frac{1}{3}}}{x-2} \text{ for } x \neq 2$$
$$= \frac{1}{5} \text{ for } x = 2, \text{ at } x = 2$$
SOLUTION

$$f(2) = \frac{1}{5} \quad \dots (given)$$

$$\lim_{x \to 2} f(x) = \lim_{x \to 2} \frac{\sqrt{x - 1} - (x - 1)^{\frac{1}{3}}}{x - 2}$$
Put x - 1 = y
 \therefore x = 1 + y
 \therefore As x \rightarrow 2, y \rightarrow 1
 \therefore $\lim_{x \to 2} f(x) = \lim_{y \to 1} \frac{\sqrt{y} - y^{\frac{1}{3}}}{1 + y - 2}$
 $= \lim_{y \to 1} \frac{y^{\frac{1}{2}} - 1 - y^{\frac{1}{3}} + 1}{y - 1}$
 $= \lim_{y \to 1} \frac{(y^{\frac{1}{2}} - 1) - (y^{\frac{1}{3}} - 1)}{y - 1}$
 $= \lim_{y \to 1} \left(\frac{y^{\frac{1}{2}} - 1}{y - 1} - \frac{y^{\frac{1}{3}} - 1}{y - 1}\right)$





$$= \lim_{y \to 1} \frac{y^{\frac{1}{2}} - 1^{\frac{1}{2}}}{y - 1} - \lim_{y \to 1} \frac{y^{\frac{1}{3}} - 1^{\frac{1}{3}}}{y - 1}$$

$$= \frac{1}{2} (1)^{\frac{-1}{2}} - \frac{1}{3} (1)^{\frac{-2}{3}} \dots [\because \lim_{x \to a} \frac{x^n - a^n}{x - a} = n. a^{n-1}]$$

$$= \frac{1}{2} - \frac{1}{3}$$

$$= \frac{1}{6}$$

$$\therefore \lim_{x \to 2} f(x) \neq f(2)$$

 \therefore f(x) is discontinuous at x = 2

Exercise 8.1 | Q 3.2 | Page 112

Test the continuity of the following function at the points indicated against them.

$$f(x) = \frac{x^3 - 8}{\sqrt{x + 2} - \sqrt{3x - 2}} \text{ for } x \neq 2$$

= -24 for x = 2, at x = 2

SOLUTION

f(2) = -24(given)
$$\lim_{x \to 2} f(x) = \lim_{x \to 2} \frac{x^3 - 8}{\sqrt{x + 2} - \sqrt{3x - 2}}$$





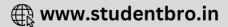
$$\begin{split} &= \lim_{x \to 2} \frac{x^3 - 8}{\sqrt{x + 2} - \sqrt{3x - 2}} \times \frac{\sqrt{x + 2} + \sqrt{3x - 2}}{\sqrt{x + 2} + \sqrt{3x - 2}} \\ &= \lim_{x \to 2} \frac{(x^3 - 8)(\sqrt{x + 2} + \sqrt{3x - 2})}{(x + 2) - (3x - 2)} \\ &= \lim_{x \to 2} \frac{(x^3 - 2^3)(\sqrt{x + 2} + \sqrt{3x - 2})}{-2x + 4} \\ &= \lim_{x \to 2} \frac{(x - 2)(x^2 + 2x + 4)(\sqrt{x + 2} + \sqrt{3x - 2})}{-2(x - 2)} \\ &= \lim_{x \to 2} \frac{(x^2 + 2x + 4)(\sqrt{x + 2} + \sqrt{3x - 2})}{-2} \quad \dots [\because x \to 2, x \neq 2 \therefore x - 2 \neq 0] \\ &= \frac{-1}{2} \lim_{x \to 2} (x^2 + 2x + 4)(\sqrt{x + 2} + \sqrt{3x - 2}) \\ &= \frac{-1}{2} \lim_{x \to 2} (x^2 + 2x + 4) \lim_{x \to 2} (\sqrt{x + 2} + \sqrt{3x - 2}) \\ &= \frac{-1}{2} \sum_{x \to 2} (x^2 + 2x + 4) \lim_{x \to 2} (\sqrt{x + 2} + \sqrt{3x - 2}) \\ &= \frac{-1}{2} \sum_{x \to 2} (x^2 + 2x + 4) \lim_{x \to 2} (\sqrt{x + 2} + \sqrt{3x - 2}) \\ &= \frac{-1}{2} \times [2^2 + 2(2) + 4] \times (\sqrt{2 + 2} + \sqrt{3(2) - 2}) \\ &= \frac{-1}{2} \times 12 \times (2 + 2) \\ &= -24 \\ \therefore \lim_{x \to 2} f(x) = f(2) \\ \therefore \text{ f(x) is continuous at x = 2} \end{split}$$

Exercise 8.1 | Q 3.3 | Page 112

Test the continuity of the following function at the points indicated against them.

f(x) = 4x + 1, for x ≤ 3
=
$$\frac{59 - 9x}{3}$$
, for x > 3 at x = $\frac{8}{3}$.





SOLUTION

$$\lim_{x \to (\frac{8}{3})^{-}} f(x) = \lim_{x \to (\frac{8}{3})^{-}} (4x + 1)$$

$$= 4\left(\frac{8}{3}\right) + 1$$

$$= \frac{32}{3} + 1$$

$$= \frac{35}{3}$$

$$\lim_{x \to (\frac{8}{3})^{+}} f(x) = \lim_{x \to (\frac{8}{3})^{+}} \frac{59 - 9x}{3}$$

$$= \frac{59 - 9\left(\frac{8}{3}\right)}{3}$$

$$= \frac{59 - 24}{3}$$

$$= \frac{35}{3}$$

$$f(x) = 4x + 1, \quad x \le \left(\frac{8}{3}\right)$$

$$\therefore f\left(\frac{8}{3}\right) = 4\left(\frac{8}{3}\right) + 1$$

$$= \frac{32}{3} + 1$$

$$= \frac{35}{3}$$

$$\lim_{x \to (\frac{8}{3})^{-}} f(x) = \lim_{x \to (\frac{8}{3})^{+}} f(x) = f\left(\frac{8}{3}\right)$$

$$\therefore f(x) \text{ is continuous at } x = \frac{8}{3}$$
Exercise 8.1 | Q 3.4 | Page 112





Test the continuity of the following function at the points indicated against them.

$$f(x) = \frac{x^3 - 27}{x^2 - 9} \text{ for } 0 \le x < 3$$

= $\frac{9}{2}$ for $3 \le x \le 6$
at x = 3

SOLUTION

f(3) =
$$\frac{9}{2}$$
(given)

$$\lim_{x\to 3} f(x) = \lim_{x\to 3} \frac{x^3 - 27}{x^2 - 9}$$
= $\lim_{x\to 3} \frac{(x-3)(x^2 + 3x + 9)}{(x-3)(x+3)}$
= $\lim_{x\to 3} \frac{x^2 + 3x + 9}{x+3}$ [As x → 3, x ≠ 3 ∴ x - 3 ≠ 0]
= $\frac{(3)^2 + 3(3) + 9}{3+3}$
= $\frac{9+9+9+9}{6}$
= $\frac{27}{6}$
= $\frac{9}{2}$
∴ $\lim_{x\to 3} f(x) = f(3)$
∴ Function f is continuous at x = 3
Exercise 8.1 [Q.4.1] Page 112
If f(x) = $\frac{24^x - 8^x - 3^x + 1}{12^x - 4^x - 3^x + 1}$ for x ≠ 0
= k, for x = 0 is continuous at x = 0, find k.





SOLUTION

Function f is continuous at x = 0

$$:: f(0) = \lim_{x \to 0} f(x)$$

$$:: k = \lim_{x \to 0} \frac{24^{x} - 8^{x} - 3^{x} + 1}{12^{x} - 4^{x} - 3^{x} + 1}$$

$$= \lim_{x \to 0} \frac{8^{x} \cdot 3^{x} - 8^{x} - 3^{x} + 1}{4^{x} \cdot 3^{x} - 4^{x} - 3^{x} + 1}$$

$$= \lim_{x \to 0} \frac{8^{x} (3^{x} - 1) - 1(3^{x} - 1)}{4^{x} (3^{x} - 1) - 1(3^{x} - 1)}$$

$$= \lim_{x \to 0} \frac{(3^{x} - 1)(8^{x} - 1)}{(3^{x} - 1)(4^{x} - 1)}$$

$$= \lim_{x \to 0} \frac{8^{x} - 1}{4^{x} - 1} \left[\left[\because x \to 0, 3^{x} \to 3^{0} \right], \left[\because 3^{x} \to 1 \therefore 3^{x} \neq 1 \right], \left[\because 3^{x} - 1 \neq 0 \right] \right]$$

$$= \lim_{x \to 0} \left(\frac{\frac{8^{x} - 1}{4^{x} - 1}}{\frac{4^{x} - 1}{2}} \right) \dots \left[\because x \to 0, \therefore x \neq 0 \right]$$

$$= \frac{\log 8}{\log 4} \dots \left[\because \lim_{x \to 0} \left(\frac{a^{x} - 1}{x} \right) = \log a \right]$$

$$= \frac{\log(2)^{3}}{\log(2)^{2}}$$

$$= \frac{3\log 2}{3\log 2}$$

$$\therefore f(0) = \frac{3}{2}$$
Exercise 3.1 [0.42] Page 112
If $f(x) = \frac{5^{x} + 5^{-x} - 2}{x^{2}} \text{ for } x \neq 0$

$$= k \qquad \text{for } x = 0 \text{ is continuous at } x = 0, \text{ find } k$$
SOLUTION
Function f is continuous at $x = 0$

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$$\therefore f(0) = \lim_{x \to 0} f(x)$$

$$\therefore k = \lim_{x \to 0} \frac{5^x + 5^{-x} - 2}{x^2}$$

$$= \lim_{x \to 0} \frac{5^x + \frac{1}{5^x} - 2}{x^2}$$

$$= \lim_{x \to 0} \frac{(5^x)^2 + 1 - 2(5^x)}{5^x \cdot x^2}$$

$$= \lim_{x \to 0} \frac{(5^x - 1)^2}{5^x \cdot x^2} \dots [\because a^2 - 2ab + b^2 = (a - b)^2]$$

$$= \lim_{x \to 0} \left(\frac{5^x - 1}{x}\right)^2 \cdot \frac{1}{5^x}$$

$$= \lim_{x \to 0} \left(\frac{5^x - 1}{x}\right)^2 \times \lim_{x \to 0} \frac{1}{5^x}$$

$$= (\log 5)^2 \times \frac{1}{5^0} \dots \left[\because \lim_{x \to 0} \left(\frac{a^x - 1}{x}\right) = \log a\right]$$

$$\therefore k = (\log 5)^2$$

Exercise 8.1 | Q 4.3 | Page 112

For what values of a and b is the function

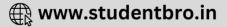
 $\begin{array}{ll} f(x) = ax + 2b + 18 & \mbox{for } x \le 0 \\ = x^2 + 3a - b & \mbox{for } 0 < x \le 2 \\ = 8x - 2 & \mbox{for } x > 2, \\ \mbox{continuous for every } x ? \end{array}$

SOLUTION

Function f is continuous for every x.

: Function f is continuous at x = 0 and x = 2

As f is continuous at x = 0.



$$\therefore \lim_{x \to 0^{-}} f(x) = \lim_{x \to 0^{+}} f(x)$$

$$\therefore \lim_{x \to 0^{-}} (ax + 2b + 18) = \lim_{x \to 0^{+}} (x^{2} + 3a - b)$$

$$\therefore a(0) + 2b + 18 = (0)^{2} + 3a - b$$

$$\therefore 3a - 3b = 18$$

$$\therefore a - b = 6 \qquad(i)$$

Also, Function f is continous at x = 2

$$\therefore \lim_{x \to 2^{-}} f(x) = \lim_{x \to 2^{+}} f(x)$$

$$\therefore \lim_{x \to 2^{-}} (x^{2} + 3a - b) = \lim_{x \to 2^{+}} (8x - 2)$$

$$\therefore (2)^{2} + 3a - b = 8 (2) - 2$$

$$\therefore 4 + 3a - b = 14$$

$$\therefore 3a - b = 10 \qquad ...(ii)$$

Subtracting (i) from (ii), we get

$$2a = 4$$

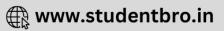
$$\therefore a = 2$$

Substituting a = 2 in (i), we get

$$2 - b = 6$$

$$\therefore b = -4$$





Exercise 8.1 | Q 4.4 | Page 112

For what values of a and b is the function

$$f(x) = \frac{x^2 - 4}{x - 2} \quad \text{for } x < 2$$

= ax² - bx + 3 for 2 ≤ x < 3
= 2x - a + b for x ≥ 3
continuous in its domain.

SOLUTION

Function f is continuous for every x on R.

 \therefore Function f is continuous at x = 2 and x = 3.

As f is continuous at x = 2.

$$\therefore \lim_{x \to 2^{-}} f(x) = \lim_{x \to 2^{+}} f(x)$$

$$\therefore \lim_{x \to 2^{-}} \frac{x^{2} - 4}{x - 2} = \lim_{x \to 2^{+}} (ax^{2} - bx + 3)$$

$$\therefore \lim_{x \to 2^{-}} \frac{(x - 2)(x + 2)}{x - 2} = \lim_{x \to 2^{+}} (ax^{2} - bx + 3)$$

$$\therefore \lim_{x \to 2^{-}} (x + 2) = \lim_{x \to 2^{+}} (ax^{2} - bx + 3) \dots [\because x \to 2 \therefore x \neq 2 \therefore x - 2 \neq 0]$$

$$\therefore 2 + 2 = a(2)^{2} - b(2) + 3$$

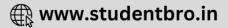
$$\therefore 4 = 4a - 2b + 3$$

$$\therefore 4a - 2b = 1 \dots (i)$$
Also function f is continuous at x = 3

$$\therefore \lim_{x \to 3^{-}} f(x) = \lim_{x \to 3^{+}} f(x)$$

$$\therefore \lim_{x
ightarrow 3^-} \left(ax^2-bx+3
ight) = \lim_{x
ightarrow 3^+} \left(2x-a+b
ight)$$





$$\therefore a(3)^{2} - b(3) + 3 = 2(3) - a + b$$

$$\therefore 9a - 3b + 3 = 6 - a + b$$

$$\therefore 10a - 4b = 3 \dots(ii)$$

Multiplying (i) by 2, we get

$$8a - 4b = 2 \dots(iii)$$

Subtracting (iii) from (ii), we get

$$2a = 1$$

$$\therefore a = \frac{1}{2}$$

Substituting $a = \frac{1}{2}$ in (i), we get

$$4\left(\frac{1}{2}\right) - 2b = 1$$

$$\therefore 2 - 2b = 1$$

$$\therefore 1 = 2b$$

$$\therefore b = \frac{1}{2}$$

$$\therefore b = \frac{1}{2}$$

$$\therefore a = \frac{1}{2}$$
 and $b = \frac{1}{2}$
MISCELLANEOUS EXERCISE 8 [PAGE 113]

Miscellaneous Exercise 8 | Q 1.1 | Page 113

Discuss the continuity of the following function at the point(s) or in the interval indicated against them. $f(x) = 2x^2 - 2x + 5 \text{ for } 0 \le x < 2$

$$= \frac{1 - 3x - x^2}{1 - x} \text{ for } 2 \le x < 4$$
$$= \frac{7 - x^2}{x - 5} \text{ for } 4 \le x \le 7 \text{ on its domain.}$$

Miscellaneous Exercise 8 | Q 1.2 | Page 113

Discuss the continuity of the following function at the point(s) or in the interval indicated against them.

$$f(x) = \frac{3^x + 3^{-x} - 2}{x^2} \text{ for } x \neq 0$$

= (log3)² for x = 0, at x = 0

SOLUTION

$$f(0) = (\log 3)^{2} \dots (given)$$

$$\lim_{x \to 0} f(x) = \lim_{x \to 0} \frac{3^{x} + 3^{-x} - 2}{x^{2}}$$

$$= \lim_{x \to 0} \frac{3^{x} + \frac{1}{3^{x}} - 2}{x^{2}}$$

$$= \lim_{x \to 0} \frac{(3^{x})^{2} + 1 - 2(3^{x})}{x^{2} \cdot (3)^{x}}$$

$$= \lim_{x \to 0} \frac{(3^{x} - 1)^{2}}{x^{2} \cdot (3)^{x}} \dots [\because a^{2} - 2ab + b^{2} = (a - b)^{2}]$$

$$= \lim_{x \to 0} \left[\left(\frac{3^{x} - 1}{x}\right)^{2} \times \frac{1}{3^{x}} \right]$$

$$= \lim_{x \to 0} \left(\frac{3^{x} - 1}{x}\right)^{2} \times \frac{1}{3^{x}}$$

$$= (\log 3)^{2} \times \frac{1}{3^{0}} \dots \left[\because \lim_{x \to 0} \left(\frac{a^{n} - 1}{x}\right) = \log a \right]$$

$$= (\log 3)^{2} \times \frac{1}{1}$$





$$egin{aligned} &= (\log 3)^2 \ &\therefore \lim_{x o 0} \, f(x) = f(0) \end{aligned}$$

 \therefore f is continuous at x = 0

Miscellaneous Exercise 8 | Q 1.3 | Page 113 Discuss the continuity of the following function at the point(s) or in the interval indicated against them.

$$f(x) = \frac{5^x - e^x}{2x} \text{ for } x \neq 0$$
$$= \frac{1}{2}(\log 5 - 1) \text{ for } x = 0 \text{ at } x = 0$$

SOLUTION

$$f(0) = \frac{1}{2}(\log 5 - 1) \dots [given]$$

$$\lim_{x \to 0} f(x) = \lim_{x \to 0} \frac{5^x - e^x}{2x}$$

$$= \lim_{x \to 0} \frac{5^x - 1 - e^x + 1}{2x}$$

$$= \frac{1}{2} \lim_{x \to 0} \frac{(5^x - 1) - (e^x - 1)}{x}$$

$$= \frac{1}{2} \lim_{x \to 0} \left[\frac{(5^x - 1)}{x} - \frac{(e^x - 1)}{x} \right]$$

$$= \frac{1}{2} \left(\lim_{x \to 0} \frac{5^x - 1}{x} - \lim_{x \to 0} \frac{e^x - 1}{x} \right)$$

$$= \frac{1}{2} (\log 5 - \log e) \dots \left[\lim_{x \to 0} \frac{a^x - 1}{x} = \log a \right]$$

$$= \frac{1}{2} (\log 5 - 1) \dots [\because \log e = 1]$$

$$\lim_{x o 0} f(x) = f(0)$$

 \therefore f is continuous at x = 0

Miscellaneous Exercise 8 | Q 1.4 | Page 113

f(x) =
$$\frac{\sqrt{x+3}-2}{x^3-1}$$
 for x ≠ 1
= 2 for x = 1, at x = 1.

SOLUTION

f(1) = 2 ...[given]

$$\begin{split} \lim_{x \to 1} f(x) &= \lim_{x \to 1} \frac{\sqrt{x+3}-2}{x^3-1} \\ &= \lim_{x \to 1} \left(\frac{\sqrt{x+3}-2}{x^3-1} \times \frac{\sqrt{x+3}+2}{\sqrt{x+3}+2} \right) \\ &= \lim_{x \to 1} \left(\frac{x+3-4}{(x^3-1)(\sqrt{x+3}+2)} \right) \\ &= \lim_{x \to 1} \frac{x-1}{(x-1)(x^2+x+1)(\sqrt{x+3}+2)} \\ &= \lim_{x \to 1} \frac{1}{(x^2+x+1)(\sqrt{x+3}+2)} \dots \text{[As } x \to 1, x \neq 1 \therefore x - 1 \neq 0] \\ &= \frac{1}{\lim_{x \to 1} (x^2+x+1) \times \lim_{x \to 1} (\sqrt{x+3}+2)} \\ &= \frac{1}{(1^2+1+1) \times (\sqrt{1+3}+2)} \\ &= \frac{1}{3}(2+2) \end{split}$$

$$= \frac{1}{12}$$

$$\therefore \lim_{x \to 1} f(x) \neq f(1)$$

$$\therefore \text{ f is discontinuous at x = 1}$$

Miscelaneous Exercise 8 | Q 1.5 | Page 113

$$f(x) = \frac{\log x - \log 3}{x - 3} \text{ for x \neq 3}$$

$$= 3 \qquad \text{ for x = 3, at x = 3.}$$

SOLUTION

$$f(3) = 3 \dots [\text{given}]$$

$$\lim_{x \to 3} f(x) = \lim_{x \to 3} \frac{\log x - \log 3}{x - 3}$$

Substitute x - 3 = h

$$\therefore x = 3 + h,$$

as x $\rightarrow 3, h \rightarrow 0$

$$\therefore \lim_{x \to 3} f(x) = \lim_{h \to 0} \frac{\log(h + 3) - \log 3}{3 + h - 3}$$

$$= \lim_{h \to 0} \frac{\log(\frac{h + 3}{3})}{h}$$

$$= \lim_{h \to 0} \frac{\log(1 + \frac{h}{3})}{(\frac{h}{3})} \times \frac{1}{3}$$

$$= \frac{1}{3} \lim_{h \to 0} \frac{\log(1 + \frac{h}{3})}{(\frac{h}{3})}$$

$$= \frac{1}{3} (1) \dots \left[\because \lim_{x \to 0} \frac{\log(1 + x)}{x - 1} = 1\right]$$

$$=rac{1}{3}$$

 $\therefore \lim_{x o 3} f(x)
eq f(3)$

 \therefore f is discontinuous at x = 3

Miscellaneous Exercise 8 | Q 2.1 | Page 113 Find k if the following function is continuous at the points indicated against them.

$$f(x) = \left(\frac{5x-8}{8-3x}\right)^{\frac{3}{2x-4}} \text{ for } x \neq 2$$

= k for x = 2 at x = 2.
SOLUTION

f is continuous at x = 2

$$\therefore f(2) = \lim_{x \to 2} f(x)$$
$$\therefore k = \lim_{x \to 2} \left(\frac{5x - 8}{8 - 3x}\right)^{\frac{3}{2x - 4}}$$

Substitute x - 2 = h

$$\therefore k = \lim_{h \to 0} \left[\frac{5(2+h) - 8}{8 - 3(2+h)} \right]^{\frac{3}{2(2+h) - 4}}$$
$$= \lim_{h \to 0} \left(\frac{10 + 5h - 8}{8 - 6 - 3h} \right)^{\frac{3}{2h}}$$
$$= \lim_{h \to 0} \left(\frac{2 + 5h}{2 - 3h} \right)^{\frac{3}{2h}}$$





$$= \lim_{h \to 0} \frac{\left(1 + \frac{5h}{2}\right)^{\frac{3}{2h}}}{\left(1 - \frac{3h}{2}\right)^{\frac{2}{5h}}}$$

$$= \frac{\lim_{h \to 0} \left[\left(1 + \frac{5h}{2}\right)^{\frac{2}{5h}}\right]^{\frac{5}{2} \times \frac{3}{2}}}{\lim_{h \to 0} \left[\left(1 - \frac{3h}{2}\right)^{\frac{-2}{5h}}\right]^{\frac{-3}{2} \times \frac{3}{2}}}$$

$$= \frac{e^{\frac{15}{4}}}{e^{\frac{-9}{4}}} \dots \left[\because h \to 0, \frac{5h}{2} \to 0, \frac{-3h}{2} \to 0 \text{ and } \lim_{x \to 0} (1 + x)^{\frac{1}{x}} = e\right]$$

$$= e^{\frac{24}{4}}$$

Miscellaneous Exercise 8 | Q 2.2 | Page 113

Find k if the following function is continuous at the points indicated against them.

$$f(x) = \frac{45^x - 9^x - 5^x + 1}{(k^x - 1)(3^x - 1)} \text{ for } x \neq 0$$
$$= \frac{2}{3} \text{ for } x = 0, \text{ at } x = 0$$

SOLUTION

f is continuous at x = 0

$$\begin{split} &\therefore \lim_{x \to 0} f(x) = f(0) \\ &\therefore \lim_{x \to 0} \frac{(45)^x - 9^x - 5^x + 1}{(k^x - 1)(3^x - 1)} = \frac{2}{3} \\ &\therefore \lim_{x \to 0} \frac{9^x \cdot 5^x - 9^x - 5^x + 1}{(k^x - 1)(3^x - 1)} = \frac{2}{3} \\ &\therefore \lim_{x \to 0} \frac{9^x (5^x - 1) - 1(5^x - 1)}{(k^x - 1)(3^x - 1)} = \frac{2}{3} \end{split}$$

$$\therefore \lim_{x \to 0} \frac{(5^x - 1)(9^x - 1)}{(k^x - 1)(3^x - 1)} = \frac{2}{3}$$

$$\therefore \lim_{x \to 0} \frac{(\frac{5^x - 1}{x^2})(\frac{9^x - 1}{x})}{\frac{(k^x - 1)(3^x - 1)}{x^2}} = \frac{2}{3} \quad ...[\because x \to 0, \therefore x \neq 0 \therefore x^2 \neq 0 \text{ Divide Numerator and Denominator by } x^2]$$

$$\therefore \frac{\lim_{x \to 0} \left(\frac{5^x - 1}{x}\right)\left(\frac{9^x - 1}{x}\right)}{\frac{1}{x^2}} = \frac{2}{3}$$

$$\therefore \frac{\log 5 \cdot \log 9}{\log k \cdot \log 3} = \frac{2}{3} \quad [\because \lim_{x \to 0} \frac{a^x - 1}{x} = \log a]$$

$$\therefore \frac{\log 5 \cdot \log(3)^2}{\log k \cdot \log 3} = \frac{2}{3}$$

$$\therefore \frac{\log 5 \cdot \log(3)^2}{\log k \times \log 3} = \frac{2}{3}$$

$$\therefore \frac{\log 5}{\log k} = \frac{1}{3}$$

$$\therefore \frac{\log 5}{\log k} = \log k$$

$$\therefore \log(5)^3 = \log k$$

$$\therefore (5)^3 = k$$

$$\therefore k = 125$$

Miscellaneous Exercise 8 | Q 2.3 | Page 113

Find k if the following function is continuous at the points indicated against them.

$$f(x) = (1 + kx)^{\frac{1}{x}}$$
, for x $\neq 0$
= $e^{\frac{3}{2}}$, for x = 0, at x = 0

SOLUTION

f is continuous at x = 0

$$\lim_{x o 0} f(x) = f(0)$$



$$\lim_{x \to 0} (1 + kx)^{\frac{1}{x}} = e^{\frac{3}{2}}$$

$$\lim_{x \to 0} \left[(1 + kx)^{\frac{1}{kx}} \right]^k = e^{\frac{3}{2}}$$

$$\therefore e^k = e^{\frac{3}{2}} \dots \left[\lim_{x \to 0} (1 + x)^{\frac{1}{x}} = e \right]$$

$$\therefore k = \frac{3}{2}$$

Miscellaneous Exercise 8 | Q 3.1 | Page 113

Find a and b if the following function is continuous at the point indicated against them.

$$f(x) = x^2 + a$$
, for x ≥ 0
= $2\sqrt{x^2 + 1} + b$, for x < 0 and f(1) = 2 is continuous at x = 0

SOLUTION

Since,
$$f(x) = x^2 + a$$
, $x \ge 0$
 $\therefore f(1) = (1)^2 + a$
 $\therefore 2 = 1 + a$ [$\therefore f(1) = 2$]
 $\therefore a = 1$

Also f is continuous at x = 0

$$\therefore \lim_{x \to 0^{-}} f(x) = \lim_{x \to 0^{+}} f(x)$$

$$\therefore \lim_{x \to 0^{-}} \left(2\sqrt{x^{2}+1} + b \right) = \lim_{x \to 0^{+}} \left(x^{2} + a \right)$$

$$\therefore \left(2\sqrt{0^{2}+1} + b \right) = 0^{2} + 1$$

$$\therefore 2\sqrt{0^{2}+1} + b = 0^{2} + 1$$

$$\therefore 2(1) + b = 1$$

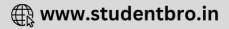
$$\therefore b = -1$$

$$\therefore a = 1 \text{ and } b = -1$$

Miscellaneous Exercise 8 | Q 3.2 | Page 113

Find a and b if the following function is continuous at the point indicated against them.





$$f(x) = \frac{x^2 - 9}{x - 3} + a, \text{ for } x > 3$$
$$= 5, x = 3$$
$$= 2x^2 + 3x + b, \text{ for } x < 3$$
is continuous at x = 3

SOLUTION

f is continuous at x = 3

$$f(3) = \lim_{x \to 3^{-}} f(x)$$

$$= \lim_{x \to 3^{-}} (2x^{2} + 3x + b)$$

$$f(3) = 5 = 2(3)^{2} + 3(3) + b$$

$$f(3) = 5 = 18 + 9 + b$$

$$f(3) = -22$$
Also, $f(3) = f(3) = \lim_{x \to 3^{+}} f(x)$

$$f(3) = f(3) = \lim_{x \to 3^{+}} f(x)$$

$$f(3) = h(3)$$

Find a and b if the following function is continuous at the point indicated against them.



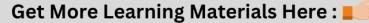
$$f(x) = \frac{32^x - 1}{8^x - 1} + a \text{, for } x > 0$$

= 2, for x = 0
= x + 5 - 2b, for x < 0
is continuous at x = 0

SOLUTION

f is continuous at x = 0

 $\therefore \lim_{x \to 0^{-}} f(x) = f(0)$ $\therefore \lim_{x \to 0^{-}} (x + 5 - 2b) = 2$ $\therefore 0 + 5 - 2b = 2$ $\therefore 5 - 2 = 2b$ $\therefore 2b = 3$ $\therefore b = \frac{3}{2}$ Also $\lim_{x \to 0^{+}} f(x) = f(0)$ $\therefore \lim_{x \to 0^{+}} \left(\frac{32^{x} - 1}{8^{x} - 1} + a\right) = 2$ $\therefore \lim_{x \to 0^{+}} \left(\frac{32^{x} - 1}{8^{x} - 1} + a\right) = 2$







$$\therefore \frac{\log 32}{\log 8} + a = 2 \dots \left[\because \lim_{x \to 0} \frac{a^x - 1}{x} = \log a \right]$$
$$\therefore \frac{\log(2)^5}{\log(2)^3} + a = 2$$
$$\therefore \frac{5 \log 2}{3 \log 2} + a = 2$$
$$\therefore \frac{5}{3} + a = 2$$
$$\therefore a = 2 - \frac{5}{3}$$
$$\therefore a = \frac{1}{3}$$
$$\therefore a = \frac{1}{3} \text{ and } b = \frac{3}{2}$$



